Chit Chat Club Essay for October 8, 2019

Seeking Knowledge about Reality: Plato, Probability, and Quantum Mechanics

D. Warner North

Version of October 28, with modest corrections, thanks to a careful reading by Chit Chat Club friend Kenneth Quandt and helpful comments from Richard Muller. Any remaining errors are the author's responsibility.

Fifty years ago I was working on my Ph.D. thesis, "The Invariance Approach to the Probabilistic Encoding of Information." I was completing a transition from graduate education in physics into the emerging field of decision analysis, built upon a combination of probability theory and economics. My thesis came out of a question asked to me in a Stanford seminar, "Why is maximum entropy the appropriate way to select a probability distribution to represent a state of information?" I believe I built a good chain of reasoning, building upon my physics background about entropy as a measure of randomness.[1] The underlying idea was invariance, that if you could not distinguish a difference between two states of information when one was a transformation from the other, you should assign the same probabilities. This principle is a generalization of the ***Principle of Insufficient Reason,*** first formulated by Jacob Bernoulli in the seventeenth century and developed by the Marquis de Laplace, in his 1814 book on probability.[2]

Let's consider simple examples of this Principle, based on symmetry. Exchange the labels on the two sides of a coin, or the six faces of a die, or 52 cards in a deck. Assuming a physical process of flipping of the coin, rolling dice, or shuffling of the cards has removed any basis for distinguishing one outcome from another, such ***an exchange of labels should leave your state of information the same***.

This is a different view from those who view probabilities as describing the ***frequency*** of outcomes in a large number of identical experiments, or those who view probabilities as ***subjective***, as measured by willingness to make or accept a wager, that is, betting odds. Such subjective probabilities may differ from person to person, and the state of information is not made obvious. As information changes, probability judgments ought to change. There is a formula for this first articulated by the Reverend Thomas Bayes in 1760. Those who view probability from the subjective perspective are usually referred to as "Bayesians."[3] Bayesian probability can be applied to one-of-a-kind events, such as whether Donald Trump will have a second term as President.[4] Bayes' Rule applies in any use of probability. It is a simple consistency check between the "prior" probability before the new information arrives, and "posterior" probability afterwards. I'll give a simple illustration in a moment. Most people in the scientific community view probability from one of these two perspectives, rather than from the invariance viewpoint I learned from Laplace and my teachers, and developed in my dissertation. Symmetry and and invariance to relabeling provide the basis for assigning probability that goes straight from a description of the state of information to the values of the probability numbers. Think about the flip of a

---

[1] In simple lay terms, entropy is lack of order, the negative of information. Maximum entropy is complete randomness. See my dissertation (North 1970) or standard physics texts. The mathematical expression is

$$H(p_1 \dots p_N) = -\sum_{k=1}^{N} p_k \log p_k$$ for N discrete intervals and p_k the probability of an outcome being in the kth interval.

[2] Laplace, 1814. Other references and development of the ideas (with the math) are in my dissertation, North 1970, and in North 2016.

[3] This viewpoint has become widespread since World War II. Earlier, the frequency basis was predominant in science.

[4] Here there is no frequency: This future event will happen, or it will not.

coin as the example. It demonstrates (on the smallest scale of two outcomes) the concept of maximum randomness.[5] You can't get more uncertain than on the outcome of a coin flip.

My physics education included statistical mechanics, which concerns the behavior of a large number of interacting entities, such as molecules of gas in a box. A Yale Professor, Josiah Willard Gibbs, had pioneered the development of this field in the late nineteenth century. His 1902 book provided a theoretical foundation for thermodynamics, just as the atomic basis for matter was being accepted by physicists. [6]

I had learned statistical mechanics from Felix Bloch, a 1952 Nobel Laureate who was a student of Werner Heisenberg, one of the founders of quantum theory. As I was leaving physics and learning about probability in other Stanford departments, I asked for a meeting with Professor Bloch in his office. When I mentioned the Bayesian viewpoint, that probabilities were not based on frequencies but could represent belief about uncertain events, Bloch accused me of heresy: "Probabilities, he stated emphatically, are frequencies in experimental data." But as I was leaving, he told me that he knew a person who had the same heretical beliefs that I had just expressed: E. T. Jaynes, who left the Stanford Physics Department shortly before my arrival. Bloch suggested I should read his papers. I subsequently learned my Ph.D. advisor knew Jaynes and admired his work. I was able to get Jaynes' unpublished lecture notes, which described his maximum entropy principle. The published version of Jaynes' book came out in 2003, after Jaynes' death. While it had a good review in *Science*[7] it is not yet widely known and accepted. My invariance ideas are not widely known and shared. But they are straightforward extensions of Laplace's concept of probability from two centuries ago.

My dissertation showed that statistical mechanics could be developed from invariance, and in particular, that the main results assuming the classical physics of Newton's Laws could be obtained without needing to make the assumption that ***frequencies in ensembles*** (that is, imagined large numbers of experimental repetitions that allow a frequency interpretation) ***equaled time averages*** in physical systems. But I did not go from the classical physics of Newton's Laws to consider the application of probability to quantum mechanics. I reasoned that I needed build a career, and not take up a position in a philosophical debate about the proper foundation of probability. I needed to get my Ph.D. finished and to put my attention into applied practical problems. I should be able to have a good career helping people in business and government to make better decisions in the face of uncertainty. (I had just helped the Mexican government's electricity authority decide to go forward toward purchasing a nuclear power reactor. This was a heady start for a young decision analyst. I did not consider trying to make a career in academia advocating new ideas. Physics seemed overpopulated with very smart

[5] If a coin is flipped N times, the most probable outcome sequence of heads and tails is N/2 for each. That is maximum randomness, or maximum entropy. Now extend the idea to more than two outcomes, in fact, go to a continuum of outcomes over multiple dimensions, such as location in space. The math is straightforward.

[6] Ludwig Boltzmann, the inventor of statistical mechanics, committed suicide in 1906, just after Einstein had published his 1905 paper on Brownian motion. Einstein's paper provided evidence of the existence of atoms and molecules that within a few years convinced skeptics that Boltzmann's theories were correct.

[7] And high praise from a few subsequent writers, such as Nassim Taleb, author of *The Black Swan*.

people. Let me get the Ph.D., my “union card,” and get going on some challenging real-world decision problems.[8]

50 years later, in my third Chit Chat Club Essay, I return to this topic I did not pursue, uncertainty in quantum mechanics, and to the epistemological questions that go with this uncertainty. These questions are now the subject of a large number of books, questioning the interpretation held by Professor Bloch and his mentor Werner Heisenberg and the other founders of quantum mechanics, and questioned earlier by Albert Einstein. The leading person in formulating this interpretation was Niels Bohr. It is called the *Copenhagen interpretation,* after Bohr’s location.

Over the past months I have reread the book I studied at Yale and many contemporary books on quantum mechanics and its relation to probability. I am bravely trying to follow Michael Thaler’s talk last month and take you on a first-person tour of what may be another scientific revolution of the type described by Thomas Kuhn (1962). This one is still in the emerging stage. I am going to do this in a short period of time, with minimum math -- and include going back to Plato.

First, let me introduce two problems with the interpretation of quantum mechanics. The first comes from “Now: The Physics of Time” (2016) by my friend Richard Muller, whose career in physics has been at UC-Berkeley. Chapter 17 has the title “A Cat Both Dead and Alive.” Quantum mechanics pioneer Erwin Schrödinger imagined an experimental set up where a quantum event, a radioactive decay measured by a Geiger counter, would release poison sufficient to kill a cat sealed inside a box. The probability that the decay will occur within an hour is one-half. “Open the box in an hour and the odds are 50 percent you’ll find a dead cat, 50% you’ll find a live one.”[9] According to the Copenhagen interpretation, uncertainty is represented by a wave function, which collapses when a measurement is taken.[10] The measurement is opening the box. What happens to the cat? According to Muller and the Copenhagen interpretation, “Until someone peeks, the cat is simultaneously both dead and alive.”

Muller then goes on to tell that he asked his wife to read this chapter: “She found the section on Schrodinger’s cat … completely incredible.” Muller goes on to say, “*But ask any physicist”* (italics in original). Yes, many physicists continue to support the Copenhagen interpretation, that reality is described by wave functions whose amplitude squared gives probabilities, and that measurements cause the wave function to collapse into a numerical measurement, in this case, resolution that the cat is either dead or alive. Quoting Muller further on in his book, “What is the situation today?” Muller’s answer: “Virtually all physicists accept the Born-Heisenberg point of view.”[11]

Two of the great physicists who were my idols in the 1960s complained about the Copenhagen interpretation. Murray Gell-Mann, responsible for the name “quark” and the “Standard Model” of

---

[8] My two most notable in the early 1970s are described in North, 2016.

[9] Muller, 2016, page 195.

[10] Muller, 2016 , page 200, describes “collapse” as a simplification. “….the wave function can be a complicated superposition of complex numbers … changing [as a result of making a measurement] to something in agreement with your measurement … a live cat or a dead cat but not both. All we ever see are the simple results of measurements, and these do not contain weird combinations like dead and alive – just dead *or* alive.”

[11] Muller, p. 196.

elementary particles, said in 1976, "Niels Bohr brainwashed a whole generation of theorists into thinking that the job [of understanding quantum mechanics] was done 50 years ago."[12] Richard Feynman, perhaps the best known physicist of his generation (whom I had the opportunity to experience personally during his frequent visits to the Stanford Physics Department) stated, "I think I can safely say that no one understands quantum mechanics."[13]

Many books are now being published questioning whether the Copenhagen interpretation needs modification. The collapsing wave function seems "completely incredible," compared to a view that the probability describes ***what we know about the cat***. Surely there should be a clearer explanation than "superposition of quantum states" for dead cat and a live cat, prior to measurement by opening the box.[14]

Now I give a second example, one that greatly troubled Albert Einstein. Can a wave function describing two particles change at a rate faster than the speed of light? Let us suppose that a gamma ray causes the creation of an electron and a positron.[15] They go in nearly opposite directions at high speed. Imagine an experiment in which the spin of the electron is measured.[16] Then, some distance away and a very short time later, the spin of the positron is measured. By conservation of angular momentum, when the spin of the electron is up, the measured spin of the positron should be down - and vice versa. But if the first measurement determines the outcome of the second, how did the positron know whether a measurement was made on the electron? Did the positron change its spin? Did the electron wave function collapse with faster-than-light communication to the positron before the second measurement? This would violate Einstein's theory of special relativity, which is accepted by virtually all physicists. Such non-local interactions, possibly involving faster-than-light travel, have been called "spooky action at a distance."

It turns out there are many ways to create pairs of particles that are ***entangled*** in this way. Pairs of electrons and pairs of photons are now the favorites. Does entanglement enable faster-than-light communication? The answer appears to be a definite no. But on the other hand, measurement of one

---

[12] Greenstein, 2019, p. 40. Source of quote is in a note on p. 132.

[13] Feynman (1965), quoted in Greenstein (2019) on the front flyleaf and on p. 120. The remainder of the quote is as follows: "So do not take the lecture too seriously, feeling you have to understand it in some model I am going to describe, but just relax and enjoy it. I am going to tell you what nature behaves like. If you will simply admit that maybe she does behave like this, you will find her a delightful, entrancing thing."

[14] Ananthaswamy (2018, p. 189) gives an updated version due to Roger Penrose of the Schrödinger cat example, with a photon going through a beam splitter that sends it through the equivalent of two doors at once. Ananthswamy adds, "As with most physicists who have trouble with quantum mechanics, Penrose finds the idea that measurement is needed to collapse the wave function implausible. … an object as large as a superimposed cat would remain in superposition for only a small fraction of a second before collapsing into one classical state. In the case of a Schrödinger's cat, Penrose's theory could cause a collapse of the total system, so the cat would be alive or dead instantaneously." (p. 190. See also Penrose, 2004, page 804-812. The term "Schrodinger cat states" was used in a recent publication in the journal *Science.* https://science.sciencemag.org/content/365/6453/570.

[15] Observing this reaction was how the positron – the anti-particle to the electron - was discovered in 1932, resulting in a Nobel Prize for the discoverer. See the Wiki for Positron.

[16] The electron has a spin and a negative charge – think of it as a tiny magnet. Spin up means the magnet has the north pole on top.

particle does seem to affect the other measurement in ways consistent with the quantum mechanical calculation. This "action at a distance" seems inconsistent with local "hidden variables" – that is, properties of the particles at the time and place of the measurements. A theorem by John Bell in 1964[17] showed that local hidden variable theories imply predictions that are different from those using the mathematics of quantum mechanics. Successes in subsequent experiments support quantum mechanics rather than the local "hidden variable" theories. Therefore, worries about the interpretation, and locality versus non-locality, are set aside in favor of using the mathematical formulation from Born and Heisenberg. That is, "Shut up and calculate."[18] Such quantum mechanical calculations have enabled modern electronics, lasers, and a lot of other aspects of modern technology. And these calculations have worked for the strong force that binds the neutrons and protons in the atomic nucleus, and for the weak force that governs radioactive decay.

Just what is quantum *entanglement?* And what is a wave function, or a spin matrix, the other (equivalent) mathematical representation for calculating probabilities in quantum mechanics? The wave function can describe a system, two entangled particles, such as an electron and positron created from the same gamma ray. As we learn the spin of the first, we should learn about the spin of the second when it is measured, unless measurement is causing information to be lost. Information gained from the first measurement cannot be communicated faster than the speed of light. But the result of the second measurement could be anticipated from the first.[19] Is the result of the second measurement **revealed** – or was it **created** from the first measurement? Or both? Spooky stuff, not just for Halloween!

What is going on here in terms of a description of what we know about reality? Quantum mechanics can give very precise information about probability distributions on repeated experiments, but not resolution of the uncertainty about uncertain quantum events, except that what is measured in one place may affect what is measured in another place – which might be very far away.

How do we learn about reality is an age-old question. Let's go back to Plato and the question of epistemic knowledge: How do we know what we know?

Plato wrote a lengthy dialogue in which the title character, Theaetetus, responds to interrogation by Socrates. It is a deep exploration into wisdom – what is knowledge? Knowledge comes from experience, ours and others, via the senses. How do we know opinion, based on perceived experience, to be correct? In order to communicate, we need language: agreed-on names and concepts. The concepts include numbers and geometry, more broadly, logic and mathematics. Different people have differing experiences. Measurement facilitates comparison. How can we tell whether we are dreaming? Are memories from past perceptions valid in the present? To what extent is there agreement with others? This reader takes Plato's lesson to be that pinning down "what is knowledge" is a tricky process,

---

[17] See the Wiki for Bell's theorem. It is really an inequality comparing the result of calculations using the mathematics of the Born-Heisenberg formulation of quantum mechanics with calculations assuming local hidden variables.

[18] While this quote has been attributed to Richard Feynman, it came from David Mermin.

[19] Spin must be measured on the same axis. It is much easier to do the equivalent experiment with entangled electrons or photons. See the next section, following the excursion into philosophy.

far more complex than just collecting perceptions. The collective sensory experience from oneself and others *must be organized in the mind*. Plato's dialogue indicates this need but offers only hints on how to do it.[20]

John Locke approached the question of what is knowledge in a similar fashion to Plato. Knowledge comes from experience, as opposed to revelation, which Locke associated with religious faith. If knowledge falls short of certainty, then call it probable knowledge. Locke did not have the quantitative concept of probability for quantitative reasoning, but only the qualitative concept. Conformity with one's own sensory experience, and testimony from others based on their experience, can improve understanding and predictability.[21]

Hume questioned whether any kind of knowledge could be validated as true. Space, matter, and causal connection are merely opinion, and potentially, delusion. Epistemic philosophy thus hit an impasse.

Kant responded to Hume's skepticism by putting in place a framework for organizing sensory experience. Space and time are basic in the sense of being inherent in all human experience: the mind imposing form on sensory content from the external world. Causation is a form of understanding, and not a reality that can be confirmed.[22]

The above is an overview of material I learned in my introductory philosophy class at Yale. Senior year Professor Henry Margenau became my tutor, using his book, *The Nature of Physical Reality*, then a decade old.

I reread this book of nearly 500 pages after reading the hundred pages of Plato's *Theaetetus* Dialogue and refreshing myself on Locke, Hume, and Kant. Prof. Margenau probed deeply into the questions of how do we know what we think we know about reality, covering much of the same ground as in Plato. It wasn't in the form of Socratic drawing-out-by-questions, but rather building up how one needs concepts, careful definitions, use of numbers and logic, assumptions that objects once seen persist in time, etc. The first hundred pages are all philosophy, followed by a chapter on empirical confirmation. Then follows a chapter on space and time, following Kant, then discussion on Newtonian mechanics and electromagnetic field theory, then a "first outline" of reality. Only then, after 300 pages does the book get into the crisis in classical physics and subsequent spooky ideas of quantum mechanics.

When at Yale I had little exposure to quantum mechanics and also to probability theory. At that time I would not have objected to Margenau's chapter subtitle, "Probability Is a Measurable Physical Quantity." This is the same viewpoint Prof. Bloch had, that **probability in physics meant frequency in the data** from running the same experiment a large number of times. Random errors in measurements

---

[20] Penrose (2004) makes an eloquent plea in the opening pages (p. 11-13, 18) for mathematics. I read Plato to be equally concerned with definitions and measurement. It is not just a matter of having the math, but learning how to apply and interpret it. Reading Penrose on Schrodinger's Cat (footnote 12) persuades me he should have read Margenau. See below.
[21]Locke, "An Essay Concerning Human Understanding," 1690, in *The English Philosophers*, p. 379. Hacking (1975) discusses Locke and notes that quantitative inductive reasoning using probability only appeared later.
[22] Kant, paraphrased in Castell, p. 209.

led to differences in the individual observations. When errors were small and independent one gets a bell-shaped curve, the "normal" or Gaussian probability distribution.[23] The average or mean value of many measurements should be a better estimate of the true value than any single measurement.

In physics lab classes I learned about systematic errors. These are often bigger, in the same direction, and from an unknown cause. Such errors make the experiment not come out with the value it should. These must be considered *epistemic* uncertainty – where we don't know, and further information will help us to learn. Random errors from statistical fluctuations are called *aleatory* uncertainty.

At Stanford I took many statistics courses dealing with aleatory uncertainty and then had a revelation: the importance of Bayes' Rule and conditional probability in the context of epistemic uncertainty. My future Ph.D. advisor, Ronald Howard, gave the following apparently simple problem on the Ph.D. exam for my new field of study. Nearly everyone taking the exam failed to get the answer. I was to take this exam the next year.

Here is the problem: A coin, with a head (H) and tail (T) and presumably "fair" in the sense of probability of head 50%, is tossed three different times. You are given the **added information** that **at least one of the tosses resulted in a head**. What is the probability that all three tosses were heads?[24]

Probability is **NOT** a measureable physical quantity, said Professor Howard. He demonstrated that by tossing a coin high in the air, catching it, and putting it down on the back of his other hand, with the catching hand on top. What is the probability of Heads, he asks the class? Then he lifts the hand so that one student, "George," in the front row, can see which side of the coin is up. What is George's probability of heads? It is either 100% or 0%, while the rest of you say it is 50%. How can the same physical event have different probabilities, and you all would agree on what these probabilities should be? It is because George and the rest of you have different states of information about the same event. Probability is NOT a physically measurable quantity, like temperature. It is in the mind, and it depends on your state of information. **This is a crucial difference from the thinking of Professors Margenau and Bloch. And it may be a crucial conceptual difference in how to think about probability in quantum mechanics. What is epistemic, and what is aleatory?** I think I understand the Schrödinger cat question as one where uncertainty is resolved at the time of the decay, and there is not a collapse of a wave function at the time the box is opened. Rather, when the box is opened is when we as observers learn the resolution of the uncertainty. But I 'm still cogitating on Bell's theorem and how measures of spin on different axes (horizontal versus vertical, or in between) might gain and lose information linking the quantum state of one particle to the state of another.

[23] The "Gaussian" name is from German mathematician Carl Fredrich Gauss. See Wiki for Normal Distribution
[24] The answer is 1/7. Write out the eight sequences of H and T for three coin tosses. HHH, THH, HTH, HHT, TTH, THT, HTT, TTT. They are equally likely with a 50% probability of H on each flip, and independence between flips. The new information rules out one sequence, three tails. Seven equally likely sequences are left. This is a simple illustration of Bayes' Rule.

Might Bayesian probability resolve the issue? There is a group of people who call themselves Quantum Bayesians.[25] But they seem to be stuck on a subjective view of probability, without apparent principles for where the subjective probabilities should come from. I take encouragement from a sentence from this September's special issue of *Scientific American* on "Truth, Lies, & Uncertainty:" "Invariants define objective truth."[26] Inference about reality should not rest on subjective judgment, such as betting odds. Maybe symmetry based on invariance can provide the basis.

Professor Margenau's 1950 book acknowledges Bayesian, or inferential, use of probability. He cannot accept it and stays with the frequency definition. But he carefully avoids endorsing the Copenhagen Interpretation with a collapsing wave function brought on by measurement. He rather takes, as his point of departure, that the experimental observations leading to quantum mechanics indicate that position and velocity cannot be measured simultaneously beyond a certain level of precision. This is the ***Heisenberg Uncertainty Principle***. It forces a change in our understanding of what we have called particles and electromagnetic fields - radio waves, light, X-rays, gamma rays. Particles are what get created and counted. But these "particles" behave like waves: they are not localized in space or in terms of how fast they move. They are rather like little clouds of vibration and whirlpools. Thinking about electrons as little balls orbiting a nucleus is misleading. Electrons and photons have spin and polarization. Spin and polarization are measured not just in one dimension but ***across two complementary dimensions,*** such as on orthogonal axes***.*** The Heisenberg Uncertainty Principle limits the ability to measure complementary quantities such as position and velocity. And it also applies to the two dimensions of spin and of polarization. If you measure them to get a 0 or 1 on one axis, you may lose knowledge about the result on the other axis. We'll come back to this point, which is central for Bell's Theorem. It is the essence of the spookiness.

The other major conclusion Margenau draws from experimental observation is that some particles cannot fit into the same state as other particles of the same type. They must obey the ***Pauli Exclusion Principle***.[27] As applied to electrons, the Pauli Exclusion Principle means that electrons go in order into concentric shells with different quantum numbers – and this describes the chemical elements of the Periodic Table, which was developed by chemists a few decades prior to quantum mechanics. Providing an explanation for the Periodic Table and for the energy of spectral lines, as electrons moved from one orbit to another, were early successes for quantum mechanics.

What physicists call particles – photons and electrons - behave like waves as well as like particles. Physicists had access to mathematics that can be applied to describe wave behavior. These include matrix methods that could be applied to discrete outcomes such as spin up versus spin down. Both formulations seem to work in describing quantum behavior, and indeed, they are claimed to be equivalent conceptually but with different mathematical methods to describe probability from the wave-like behavior.

---

[25] Wiki, Quantum Bayesianism. See also Musser, 2019, and Ananthaswamy, 2018.
[26] Musser, 2019, p. 32.
[27] Named for physicist Wolfgang Pauli, who formulated it in 1925.

We return now to Schrödinger's cat and spooky action at a distance. First, the cat is alive or dead inside the box, but we do not know which. If we view probability as reflecting our information instead of a wave function – which is assumed as a concept but is not in any way directly measurable - then we **learn** about the state of the cat when the box is opened. The uncertain event of the radioactive decay triggering the death of the cat either happened earlier, or has not yet happened at the time the box was opened. Aleatory uncertainty describes radioactive decay, and the probability distribution describing the statistical frequency in radioactive decay has been experimentally verified to very high precision.

Now consider the second situation, an electron and positron created with opposite spins. The spin on the second particle will be the opposite from the spin on the first, by conservation of angular momentum at the pair's creation. The wave function is for the system of both particles, and aleatory and epistemic uncertainty are mixed up. We learn by measuring one particle, but then we lose information through the measurement. Can these two effects be disentangled?

The situation is easier to address experimentally with pairs of electrons or photons, where polarization is the equivalent of spin. We'll use photons, and we will consider linear polarization, separating the two components into horizontal and vertical, perpendicular to the direction of travel. 3-D movies use polarizing filters to separate two images, one for each eye.

Let's assume we have light that has been though a filter that lets though the vertical polarized photons and blocks that horizontal. If we have two polarized sheets then when they are both vertical, light goes though both. Think of the light as now being vibrations up and down, but interaction with the molecules in the filter have blocked the horizontal vibrations. Now turn one of the filters 90 degrees. Now these crossed filters block the light in perpendicular directions, and no light gets though.

Suppose we have a third sheet, and we insert it at a 45 degree angle between the first two sheets. What happens? Some light now comes through! How could this happen?

With two filters in the vertical position we are asking the question, is the photon vertically polarized? Answer: yes. With the second filter horizontal, we learn that photons that are vertically polarized are absorbed and do not pass the second filter.

But the inserted 45 degree filter takes some of the linearly polarized photons and changes them. More accurately, these photons get repolarized to 45 degrees and 135 degrees. Half of them go through the second filter. The other half are absorbed. The third filter repolarizes again at 90 degrees, and half of the 45 degree group become horizontally polarized. These make is the dim light we see coming though all three filters.

We are measuring "spin" on two axes, and the rule is that our measurement can have only two outcomes, in this case as "pass" and "don't pass." Think of switches that are either "on" or "off". After passing through the first filter, all the remaining photons were in "pass" orientation for the vertical direction – and passed though a second vertical filter. But for the 45 degree filter, we have a mixture created by the new polarization measurement. 50% of the photons will go through and 50% will be absorbed. Another 45 degree shift yields more repolarization, with another 50% of the photons absorbed.

The incoming photons from the first to the second filer and from the second to the third are repolarized. We have a combination of a randomizing process and a process of transmitting information that is not random.

This is the "aha" moment. There **are two dimensions for the quantum uncertainty.** When these are combined through measurement on an intermediate axis (that is, an angle **between** zero and 90 degrees from the vertical) some of the information is lost, and some is retained. The rules of quantum mechanics tell us that polarization (or spin) has to be considered not as a scalar number with two possible values, but as **a vector with two components**. Measurement is limited to two outcomes, here "pass" and "don't pass." (With electrons or atoms, these results can be "spin up" and "spin down.")

The angle between the measurements determines the response. This is **entanglement**, a relation between the probabilities. Angles such as 45 and 120 degrees give deviations from randomness: Some of the information is preserved. Such I**ntermediate angles are the basis for Bell's Theorem and for quantum computing.** The element for computing is a qubit, and we just demonstrated how to make them, with intermediate angle measurements.

The latest issue of *The Economist* has an article titled "Schrödinger's Cheetah," about progress in quantum computing. Regular computing is done with bits. The values are 0 and 1. Quantum computing is done with qubits. Think, spin up or spin down for an electron, or polarization (vertical or horizontal) for a photon – but in two dimensions, related by the angle. When a qubit is measured, the result is a 0 or 1 - and the measurement can change the future value of the qubit. Measurement can trigger the Heisenberg Uncertainty Principle and add randomness, but with some order preserved. The math is high-school level, but counterintuitive, if you are used to regular computing. Trigonometry functions and the Pythagorean Theorem play a large role. Probabilities of the 0 and 1 other than 50% can be designed.

120 degree angles are an important special case. By using a 120 degree angle the probability distribution can be raised to ¼ and ¾ for 0 and 1. With the probability raised to this value from 50% (maximum randomness as in coin flipping), significant information can be transferred with proper coding. My reference, *Quantum Computing for Everyone*, (MIT Press) has details and an illustration of Bell's inequality for the comparison of the quantum mechanical calculation is compared to a calculation with hidden local variables.[28] The calculated quantum mechanical answer verified by experiments differs from the one with assumed local causality by a small amount, 1/18, about 5%.

[28] Dirac's matrix notation and the rules for quantum computing are clearly explained in Bernhardt, 2019.

Confused? Do we need a paradigm shift? Here are more quotes. “Truth can be elusive even in the best-established theories. Quantum mechanics is as well tested a theory as can be, yet its interpretation remains inscrutable.”[29]]

“The trouble with quantum mechanics [is that] … it doesn’t really make sense.”[30]

Is the interpretation of probability part of the problem? Is probability a state of mind, changing as information changes, or a state of things, defined by frequencies in data - or an unobservable wave function?

What seems weird, but is an essential part of quantum mechanical calculations, is that the so-called wave function does not describe one particle. It describes the system - which might be one particle, a pair of particles once close together and now far apart, or the whole universe full of these particles that also behave like interacting waves.[31]

Can symmetry be a basis for the probabilities? Let’s consider the radioactive decay of a neutron to a proton and an electron, or, a uranium atom with the emission of an alpha particle.[32] In either case, the process has no memory: Given the decay has not yet happened, the probability of decay in the next increment of time stays the same. That makes the probability distribution for time to decay a simple exponential function, and therefore the number of decays over time is a Poisson process. These related processes are fully described by one parameter, which can be taken as the half-life. Is the process is uniform over space – all increments of solid angles (or latitude and longitude increments on a surrounding “globe”) are equally likely to contain the emitted ejected alpha or beta particle?[33]

Like a tossed coin, or an electron with spin up or spin down, we have a symmetry argument for assigning probabilities. In the absence of further information, we can assign the same probability to increments of time and to directions into space. There seems no way we can resolve this uncertainty with measurements, especially for the beta decay of the neutron. We can’t get inside with any kind of probe to find out when it is going to “give birth” and in which direction the emitted particles will come out.[34]

---

[29] From the September *Scientific American* lead article, Musser, 2019, page 30.

[30] Roger Penrose, a brilliant British mathematical physicist, who co-authored a book with Stephen Hawking, quoted in Ananstawamy, 2018, p. 189.

[31] In the interest of shortening I have placed the discussion of the Many Worlds concept due to Hugh Everett an appendix.

[32] The first is an example of the weak force. The second is a quantum tunneling effect involving the strong force and the electromagnetic force. See the Wiki, Alpha decay, and Wiki, Beta Decay

[33] Richard Muller tells me this sentence is not true. Spin has an impact. Parity violation occurs with the weak force. It is not true that the laws of nature remain the same under mirror reflection. See the Wiki for Weak interaction, “Violation of Symmetry” section. The parity violation of symmetry would need to be included in the calculation of the probabilities for the directions of the emitted particles.

[34]Possibly with the emission of a photon from an atom or an alpha particle we can image a bulge in the electron cloud or the shape of the nucleus, but the Heisenberg Uncertainty Principle may make it difficult or impossible to

For an individual decay event, we can only "see through a glass darkly" And that darkened glass is the Heisenberg Uncertainty Principle.

There is a literature on Quantum Bayes (evolved into "QBism") with subjective probabilities. Where do the subjective judgments come from? If these probabilities can be based on symmetry and physical laws, the approach seems more promising. There ought to be a basis for the probabilities that can be generally agreed-upon, rather than having them depend on human judgment, which is often wrong.

Physicists would like to think they can describe reality. But that may be "unrealistic" – to make a pun. When I began this investigation I thought that quantum mechanics should be considered a framework to describe what we can learn about physical reality through the dark glass of the Heisenberg Uncertainty Principle.

As I neared the end of preparing this essay, I realized that I may still be trapped in classical physics thinking. Don't keep thinking about individual particles. Think that the wave function adopted from the math inventory describes the whole system – from one or two entities, to the whole observable universe more than ten billion light years in diameter, and it's a concept in the mind. The wave analogy should be to a vibrating string, or to the surface of water – a quiet pond, or stormy ocean. There are peaks and valleys, and nodes with no amplitude. Think harmonics on a vibrating string – half the string vibrating, or a third, fourth, fifth, etc. Wave patterns are superimposed but we learn to recognize them – as we do musical notes and overtones, colors in light, and, more generally, frequencies in the electromagnetic field. And the vibrations include spins, like the little whirlpool you see in the water as the bathtub drains.

What carries these vibrations? Physicists in the late 19th century imagined a "luminiferous ether" as the carrier of light. But then the Michelson-Morley experiment found that the speed of light was constant despite the motion of the observers. Einstein's special relativity showed that space and time are linked into a four-dimensional structure, where time slows down for something moving fast, compared to time for the stationary observer. Consider that the vibrating medium is space-time, and that the vibrations, laterally and in terms of spinning motions, are what we at the macroscopic level perceive as both particles and waves – the duality described with quantum mechanics. But it is a vibrating medium that we are describing with our wave mathematics. This medium exists as a property of space-time in a vacuum. It is not the separate entities corresponding to photon or electrons doing the vibrating. It is like seeing ripples on a pond, and thinking the ripples are real, rather than manifestations of a wave

---

measure this precursor change without affecting the emission. Greenstein (2019) avoids such possible complexities, stating (p. 42), "In terms of radioactive decay, if there is a reason, quantum theory does not give it to us. Just as the theory has no way within its language to give a precise prediction of the direction of spin, it has no way to predict when any given nucleus will decay. IF you are looking for a reason, you are looking for a hidden variable." And, so far, none have been found. Randomness gives us precise predictions, but they are statistical, of an aleatory uncertainty that can be precisely described in mathematical terms.

affecting the medium of water. The ripples make moving patterns. The molecules of water are the carriers for these patterns.

Amplitude squared represents where the action is, in terms of probability, as waves propagate through a medium. And in quantum mechanics, because we observe only through the equivalent of other waves, when we measure we distort the pattern. Think of a dock at a lake distorting waves made by a passing boat.

Where do we find experimental evidence that should convince us to give up our thinking that particle-like behavior should dominate cause and effect – that is, that locality should be inherent in how reality should work?[35] Consider the two-slit experiment, which demonstrates that both electrons and photons can appear to go "Through Two Doors at Once," the title of a 2018 book by Anil Ananthaswamy. Yes, experiments have now demonstrated that a single photon or electron, passing through a double slit (two small vertical openings with a barriers outside and between them) appears to go through ***both***, rather than only one. It is hard to imagine better evidence of the duality of particle and wave behavior for single particles. **What we are calling a particle with wave-like behavior must be capable of being in, or transiting through, two places at once!** Can this experimental finding be reconciled with local causality? On near-local causality?

Ananthasmany's book does not make a case for one interpretation of quantum mechanics versus another. It might be viewed as offering analogies and tools that could provide a better perspective. Perhaps we need to reject that we can achieve a single perspective, and keep using competing perspectives.

Physicists have proposed quantum mechanics as a description of reality. I believe it makes more sense as a descriptive framework for gaining knowledge about reality. We cannot make observations except within the limitations of the Heisenberg Uncertainty Principle – "For now, we see as through a glass darkly."[36] Perhaps this limitation makes what we have an epistemic theory, that is, quantum calculations reflect our knowledge about reality and not reality itself.

Can this limitation be removed? Are there realities other than physical reality? Margenau is careful to limit his book to physical reality. John Locke discussed revelation, as opposed to knowledge gained through experience.

Do we have a hybrid, because we do not yet have the proper concepts to understand entanglement? I went into this investigation wondering if Bayesian probability based on symmetry and invariance, rather

[35] Ananthaswamy, p. 252. I'll add my hunch that non-locality might hold for the electromagnetic field, but not for the strong and the weak force, which act only at short distances such as inside an atomic nucleus (strong force) or inside a neutron or proton (weak force). I have not seen this conjecture that I describe as my hunch in print. Measurements such as described in this essay are made with the electromagnetic force.
[36] From I Corinthians 13:12, King James version.

than subjective probability or the frequency interpretation in repeated measurements, might help sort out the confusion. Reflecting on Plato and Margenau, I quote from George Greenstein's 2019 book, *Quantum Strangeness*, "Our language forces on us a certain way of thinking, a way that apparently we must be careful to resist." A page later he quotes the "brilliant physicist" – his term - E.T. Jaynes, my guide 50 years ago for my dissertation research into the use of probability based on invariance from symmetry.

> …it is pretty clear why present quantum theory not only does not use – it does not even dare to mention – the notion of a "real physical situation." Defenders of the theory say that this notion is philosophically naïve, a throwback to outmoded ways of thinking, and that recognition of this constitutes deep new wisdom about the nature of human knowledge. I say that it constitutes a violent irrationality, that somewhere in this theory the distinction between reality and knowledge of reality has become lost … [37]

Is there really a "violent irrationality" here? Is Jaynes expressing frustration that he did not find a single perspective, such as Bayesian probability, that provided clarity for him? Is there an improved way to reconcile Bell's theorem and locality?[38] Experimental evidence obtained over the last several decades supports Bell's theorem rather than local hidden variables. At least some of the Quantum Bayesians (QBists) disagree with those who claim that quantum mechanics is non-local. Within this group there appears to be disagreement. Improved clarity seems desirable. At this time I have not invested the time to understand the disagreements.[39] I support the Born-Heisenberg mathematics. Could there be support that quantum mechanics is correct and local if "local" is properly interpreted? I believe the interpretation of the probabilities could be explained more clearly than has been done with the language of the Copenhagen interpretation. I am inclined toward the ideas of the Quantum Bayesians, but I would like a better basis for the probabilities than subjective judgment by an agent. There ought to be principles for obtaining the probabilities that can be widely accepted as consistent with the state of information available from the collection of experimental observations by the scientific community. I believe that was what Jaynes was seeking as relief from the "violent irrationality" that troubled him.

Through experimental physics we have learned a great deal about the nature of reality, but much mystery remains. Does a measurement *reveal* a property of the microworld, or does it *create* that property, perhaps under the influence of other measurements? Or is this language of "reveal" and "create" inadequate to describe the knowledge we gain – and lose - through experiments?

---

[37] Greenstein, 2019, p. 96, from a 1980 book chapter. A later Jaynes quote (Quantum Bayesianism Wiki, top of History and development section), from a 1990 paper, is as follows: … [quantum mechanics is a] "peculiar mixture describing in part realities in Nature, and in part incomplete human information about Nature – all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble."

[38] Jaynes complained in 1989 about Bell's theorem, that there were two improper hidden assumptions regarding probability. But Jaynes seemed to accept a rebuttal by Steve Gull at the same meeting. Many commentators view Bell as having shown that quantum mechanics and local realism are incompatible. See https://en.wikiversity.org/wiki/Bell%27s_theorem/Proofs.

[39] See Quantum Bayesianism Wiki, especially History and development, and Reception and criticism sections.

We are limited by the Heisenberg Uncertainty Principle. Radioactive decay seems an excellent illustration. A nucleus emits an alpha particle early, or late, compared to the half-life, and we have no way to predict which it will be. But over repeated measurements, our data confirm a simple probability distribution that is precise to the limits of our ability to measure.

The old paradigm is the Copenhagen interpretation. It is criticized for lack of clarity. The calculations using the mathematics of waves seem to work - sometimes after a great deal of work figuring out how to do them. The many recent books I have read indicate dissatisfaction with Copenhagen interpretation and a search for a better paradigm. Ananthaswamy's, one of best of the books, in my opinion, concludes, "… there is no way to classify these theories in a consistent manner. It's a strong clue that our understanding of the quantum world is still up for grabs."[40]

Maybe I should make further investigation my retirement project. But preparing this paper has been much harder than I thought. I'll have to learn and relearn the applicable math, or find someone who already knows it. This essay represents a reconnaissance effort to see if anyone else already has a new and persuasive paradigm. I have not yet found it. Maybe a reworded Copenhagen interpretation comes close. Drop "collapsing wavefunction," and make Schrodinger's cat state a matter of what information is available to the observer.

Many capable people with strong backgrounds in physics are working on improving upon the Copenhagen interpretation. Maybe we need someone young, with a fresh viewpoint, such as Einstein had a century ago on relativity.

Three Appendices for the More Technically Oriented:

Appendix 1: The "Standard Model" for Elementary Particles

Beginning about the time I was at Yale, physics came to understand the strong nuclear force, which binds together the neutrons and the protons in the nucleus. Each is made up of quarks bound together with gluons. Another force, known as the weak force, governs radioactive decay. These three forces are all well-described by quantum mechanics. There is now a "Standard Model" of so-called elementary particles, made up of hadrons (six kinds of quarks, combinations as protons, neutrons, and various mesons) and leptons (electrons and several kinds of neutrinos), and the bosons (photons, gluons, mesons, and other particles with spin values of a whole number). Fermions have half integer spins. The Pauli Exclusion Principle applies to fermions and not bosons. Some atoms are bosons, such as Helium-4 (two neutrons, two protons). Some, like helium 3 (only one neutron) are fermions. With the discovery of the Higgs boson, the "Standard Model" composed of quantum mechanics and this set of elementary particles related to the three forces, may be complete in the sense that future particles found with yet-higher energy accelerators will be considered combinations of the presently known ones. Gravity, the fourth force in nature, is described by Einstein's General Theory of Relativity. No one has yet figured out

[40] Ananthasamy, page 262.

how to combine quantum mechanics and General Relativity. And no one has yet figured out how to explain observations on dark matter and dark energy. These are remaining challenges. In this essay I have focused on quantum mechanics, which has been spectacularly successful in describing what we know about the behavior of the all the known particles under the three forces, excepting gravity.

Appendix 2: The Many Worlds Interpretation

Let's look at another possibility: the "Many Worlds" interpretation from Hugh Everett, III. He was a student at Princeton a few years before I was a student at Yale. He wrote a Ph.D. dissertation that was poorly received by Niels Bohr, who developed the Copenhagen Interpretation. Everett left physics to do applied research. I interacted with him early in my similar career, but never discussed quantum mechanics. He lived hard, drank profusely, and died at 51 in 1982. Only recently has his "many-worlds" theory been given much attention (See Carroll, 2019).

In Everett's view every time uncertainty is resolved with an event like an alpha or beta decay or a spin/polarization measurement, it creates a branching of what happened from what might have happened. Think, I tossed a coin, and I got an outcome of heads, or tails. Then that outcome defines the universe in which I am located, compared to one that I might have been in. Now consider a sequence of head and tail outcomes – a giant wave function that describes the universe and its contents. The dimensionality is vast, far beyond our ability to make computations. And as each one of this vast multiplicity of quantum event uncertainties is resolved, it defines another universe - besides the one we are in. Our experience is confined to just one universe, and we have no communication with the many others. I am not sure I see the assumption of a vast number of many alternate universes, defined by the outcomes that did not happen in ours, is useful except for illustrating ongoing resolution of uncertainty on a colossal scale.

I think of the Norns, in the Prologue to Wagner's Opera, Götterdämmerung, weaving the rope of destiny – is each thread an outcome of a quantum event?[41]

Appendix 3: Entropy and Randomness

Here is a bit more explanation of entropy, maximum entropy, building upon the work of E.T. Jaynes and leading to my dissertation results. Maximum entropy is finding the most likely pattern among a large number of outcomes. Coins and dice may help us to get the general idea, without getting into the math. Consider head-tail sequences - n tosses of a coin. The binomial expansion gives 2^n sequences heads and tails. For example, for n=4, we get 16 sequences, (1 all Heads, 4 one T three H, 6 two H two T, 4 one H three T, 1 all T). Most likely are the sequences with half H and half T; one can get these six ways in the 4 coin tosses example. This maximum at n/2 holds as n gets large. We can generalize this finding of most likely arrangements for very large numbers (See North, 2016, my Springer paper) and get the entropy

[41] Wagner's reference to three Norns weaving the tapestry of fate comes from old Norse legends, recounted in poetry by medieval bards.

measure. The Bernoulli (two outcome) probability distribution has only one number, the probability of heads p, with the probability of tails being 1-p. The entropy function for this distribution has its maximum at p= 50%. Any other probability value gives less entropy – less randomness.

Maximize entropy over the classical mechanical description of positions and momenta for all the molecules in a confined volume of (ideal) gas. (This involves about 6 x10^{24} variables!). There is a constraint on the total energy in the box, and assume that the particles are identical in terms of what we know about their positions and velocities. Maximize the entropy with fixed total energy and the confining box at rest. One obtains the normal distribution for velocities, and an exponential distribution for energies of the molecules. The idea is to determine the most likely arrangement that is consistent with the known average energy. We find the probability distribution over states of the system (composed of about 10^{24} moving bodies!) that is "maximally random" consistent with the conservation principles of physics.

My dissertation showed this reasoning duplicated the main results for the probability distributions of classical statistical mechanics. Families of distributions from maximizing entropy subject to known constants yields exponential family, with property that updating with prior-posterior analysis via Bayes' Rule leads to the same "conjugate prior" family. Equivalently, these probability distributions have "sufficient statistics" – all you need to know for revising the probability distribution based on more information are sample summary statistics like mean, variance, etc. Given consistency with these summary statistics, all microstates (i.e., position, velocity) of the constituent particles are possible. We find most likely microstate arrangement, subject to conservation laws that correspond to the sufficient statistics (energy; "box at rest,"). The concept of "equilibrium" means randomizing the time of experiment adds no additional uncertainty – one get the same probability distribution back after randomizing! So this probability distribution is **invariant** to randomization over time.

This finding eliminates need for an ergodic hypothesis – that the average over large number of conceptual experimental trials (Gibbs Ensemble average over microstates of position and momenta in phase space) equals the time average.

Extending statistical mechanics into quantum mechanics using Bayesian probability on quantum states, including exclusion from the Pauli Principle, ought to be straightforward. I have not found it, or done it. Maybe it is there in the literature, but not in the philosophically oriented books I have been reading. It seems to me the insights I found for classical statistical mechanics ought to carry over. But maybe entanglement adds substantial difficulty. I can't claim to fully understand Bell's theorem. But I have not found it to be flawed.

Schrödinger's Equation is like Newton's Laws of motion in that it is deterministic in its prediction for changes in time. No extra uncertainty is added as time increases. Laplace imagined a demon, who, if provided with the initial conditions, could calculate the future paths of many interacting bodies in motion, for all future time. With Newton's laws, there is no added uncertainty from what happens over time: acceleration and therefore future paths are determined by force divided by mass for each body. Does the Laplace demon concept also apply with quantum mechanics? If given the initial conditions (the wave function or quantum state) the demon should be able to predict where the system will go over all future time. Is this true for a wave function representing a system of many interacting particles, as with Newtonian mechanics? We get information out from wave function, whose amplitude squared represents probabilities. The wave function itself is not observable. It is a concept built up with math. Using it following a set of rules gives accurate predictions for systems for which we have learned to do the calculations. The probability distributions should be invariant over randomizing the time of a measurement for equilibrium systems.

Now progress is being made on using these rules for quantum computing using 50 or so atoms (The Economist, "Schrodinger's Cheetah"). Each atom has a spin and we can encode information as qubits with entanglement – with loss of information in the readout/measurement process. That's a long way from the reality we experience in macroscopic phenomena. But it is part of the brave new world that we seem to have as our future.

REFERENCES

"Quantum computing: Schrödinger's Cheetah," *The Economist,* 71-72, September 28, 2019.

The English Philosophers from Bacon to Mill, edited by Edwin Burtt, New York: Modern Library, 1939.

Ananthaswamy, Anil, *Through Two Doors at Once: The Elegant Experiment that Captures the Enigma of Quantum Mechanics*, New York, Dutton, 2018.

Becker, Adam, *What is Real? The Unfinished Quest for the Meaning of Quantum Physics*, New York: Basic Books, 2018.

Bernhardt, Chris, *Quantum Computing for Everyone*, Cambridge, MA: MIT Press, 2019.

Castell, Alburey, *An Introduction to Modern Philosophy in Six Philosophical Problems*, New York: MacMillan, 1943.

Feynman, Richard, The Character of Physical Laws, Cambridge, MA: MIT Press, 1965.

Gibbs, Josiah Willard, *Elementary Principles in Statistical Mechanics*, New Haven, CT: Yale University Press, 1902. Reprinted by Dover Publications, New York, 1960.

Greenstein, George, *Quantum Strangeness: Wrestling with Bell's Theorem and the Ultimate Nature of Reality*, Cambridge, MA: MIT Press, 2019.

Hacking, Ian, *The Emergence of Probability: A Philosophical Study of Early idea about Probability Induction and Statistical Inference*, Cambridge, UK: Cambridge University Press, 1975, 2nd edition, 2006

Jaynes, E.T., *Probability Theory: The Logic of Science*, edited by G. Larry Bretthorst, Cambridge, UK: Cambridge University Press, 2003.

Kuhn, Thomas, *The Structure of Scientific Revolutions*, Chicago: University of Chicago Press, 1962. 50th Anniversary edition, 2012.

Laplace, Pierre-Simon, Marquis de*. Essai philosophique sur les probabilités,* 1814. English translation, *A Philosophical Essay on Probabilities,* New York, Dover, 1951.

Margenau, Henry, *The Nature of Physical Reality: A Philosophy of Modern Physics*, 1950. Reprinted by Ox Bow Press, Woodbridge , Connecticut, 1977. Available online at: https://archive.org/details/TheNatureOfPhysicalReality/page/n1.

Musser, George, *Spooky Action at a Distance*, New York: Scientific American, 2015.

Musser, George, "Virtually Reality," *Scientific American,* Special Issue on Truth, Lies and Uncertainty, **321**:30-34, September 2019. .

North, D. Warner, *The Invariance Approach to the Probabilistic Encoding of Information*, Ph.D. Dissertation, Stanford University, March 1970. Available at: http://www.northworks.net/w_pub.htm. (Click on the link near the top of the page to download.)

North, D. Warner, "Decision Analytic and Bayesian Uncertainty Quantification for Decision Support," Handbook of Uncertainty Quantification, R. Ghanem et al., eds., Switzerland: Springer International Publishing, 2016. (available from Springer online, or on request from author).

Penrose, Roger, *The Road to Reality: A Complete Guide to the Laws of the Universe*, New York: Vintage Books, 2004.

Plato*, The Philosophy of Plato*, selected and edited by Irwin Edman, 3rd edition of the Benjamin Jowett translation, New York: The Modern Library (Random House), 1956.

Smolin, Lee, *Einstein's Unfinished Revolution: The Search for What Lies Beyond the Quantum*, New York: Penguin Press, 2019.

Wikipedia, https://en.wikipedia.org/wiki/Alpha_decay. Last accessed October 7, 2019.

Wikipedia, https://en.wikipedia.org/wiki/Bell%27s_theorem. Last accessed October 2, 2019.

Wikipedia, https://en.wikipedia.org/wiki/Beta_decay. Last accessed October 7, 2019.

Wikipedia, https://en.wikipedia.org/wiki/Weak_interaction, Last accessed October 24, 2019.

Wikipedia, https://en.wikipedia.org/wiki/Ludwig_Boltzmann. Last accessed October 3, 2019.

Wikipedia, https://en.wikipedia.org/wiki/Luminiferous_aether. Last accessed October 3, 2019.

Wikipedia, https://en.wikipedia.org/wiki/Normal_distribution. Last accessed October 3, 2019.

Wikipedia, https://en.wikipedia.org/wiki/Positron. Last accessed October 3, 2019.

Wikipedia, https://en.wikipedia.org/wiki/Quantum_Bayesianism. Last Accessed October 25, 2019.

Wikiversity, https://en.wikiversity.org/wiki/Bell%27s_theorem/Proofs. Last accessed October 25, 2019.